

1 Impact Analysis of a Falling Object

The following assumptions are considered for the following analysis. The object weight is $m = 0.15\text{kg}$ (or 150g), the acceleration due to gravity $g = 9.81\text{m.s}^{-2}$, and finally the height from which it falls $s = 1\text{m}$.

From the equations of linear motions it can be calculated as follows,

$$s = ut + \frac{1}{2}at^2 \quad (1)$$

$$t = \sqrt{\frac{2s}{a}} \quad (2)$$

The time taken for the object to fall from a height of 1m, and it is only after this time has passed that the object experiences 'impact', is calculated as follows,

$$t = \sqrt{\frac{2}{9.81}} = 0.45 \text{ s} \quad (3)$$

The *impact velocity* maybe obtained by equating *Kinetic Energy = Potential Energy*,

$$\begin{aligned} \frac{1}{2}mv^2 &= mgh \\ v &= \sqrt{2gh} = 4.43 \text{ m.s}^{-1} \end{aligned} \quad (4)$$

or, since,

$$v = a \cdot t = 9.81 \cdot 0.45 = 4.43 \text{ m.s}^{-1} \quad (5)$$

Thus, the Kinetic Energy just before impact is,

$$KE = \frac{1}{2}mv^2 = 1.47 \text{ Nm} \quad (6)$$

The Work-Energy Principle states that, “*The change in the kinetic energy of an object is equal to the net work done on the object.*” For a straight-line collision, the net work done is equal to the average force of impact times the distance traveled during the impact,

Average impact force x distance traveled = change in kinetic energy

This can be mathematically described as follows,

$$F_{Avg} \cdot d = \Delta KE = KE_{Final} - KE_{Initial} \quad (7)$$

Hence,

$$F_{Avg} = \frac{1.47}{d} = \frac{1.47}{0.01 \text{ mm}} = 14,700.00 \text{ N} \quad (8)$$

From this, the deceleration (denoted by the negative sign),

$$a = -\frac{F}{m} = -\frac{14,700}{0.15} = -98,000.00 \text{ ms}^{-2} \rightarrow \frac{98,000}{9.81} = 9,989.81 \text{ g} \quad (9)$$

From $a = (v - u)/t$, the duration of the deceleration is,

$$t = \frac{(v - u)}{a} = \frac{4.43}{98,000} = 0.02 \text{ ms} \quad (10)$$

Putting this into perspective, we can consider the power generated. It is known that $KE = 1 \text{ J} = 1 \text{ Nm}$ and that $1 \text{ hp} = 745.7 \text{ W}$. Since $1 \text{ W} = 1 \text{ J/s} = 1 \text{ Nm/s}$,

$$\frac{1.47}{0.02 \times 10^{-3}} = 73,500.00 \text{ Nm/s} \rightarrow \frac{73,500}{745.7} = 98.57 \text{ hp} \quad (11)$$

Repeating the above calculation with $d = 0.001 \text{ mm}$,

$$F_{Avg} = \frac{1.47}{d} = \frac{1.47}{0.001 \text{ mm}} = 1,470,000.00 \text{ N} \quad (12)$$

Thus the acceleration is $a = -9,800,000.00 \text{ ms}^{-2}$ with a duration of $t = 452.04 \text{ ns}$. Thus,

$$\frac{1.47}{452.04 \times 10^{-9}} = 3,251,918.74 \text{ Nm/s} \rightarrow \frac{3,251,918.74}{745.7} = 4,360.9 \text{ hp} \quad (13)$$

From the above analysis it can be seen that dropping an object (with a mass of 150 g) from a height of 1 m (or $\sim 3 \text{ ft}$) will cause it to experience a shock of $\sim 10,000g$ whilst generating $\sim 100 \text{ hp}$ of power, if the distance travelled during impact is $\simeq 0.01 \text{ mm}$.

If the distance travelled during impact is reduced to $\simeq 0.001 \text{ mm}$, i.e. a harder surface like a marble or granite floor, this will cause the object to experience a shock of $1 \times 10^6 g$ and absorb $\simeq 4,400 \text{ hp}$.